Solving a Quantitative Problem

Problem Statement: The most powerful nuclear weapon ever used was the Tsar Bomba, a hydrogen bomb detonated by the Soviet Union in 1961. It had a 50 megaton yield and vaporized the island in the Arctic on which it was tested. How massive would an object that impacts the Earth at escape velocity (about 11 km s^{-1}) need to be such that the energy released equals the yield of the Tsar Bomba? You can use the fact that 1 kiloton of TNT is equal to 4.184 $\times 10^{12}$ J.

1. **Identify Known and Desired Quantities:**

   Carefully read the statement of the problem. What does it ask for and what do we know?

   - E = 50 megatons
   - v = 11 km s^{-1}
   - m = ?

2. **Pick an Equation that Relates the Known and Unknown Quantities**

   Look in the book, the lecture notes, or even better your memory to find a relationship between the given and desired quantities. In this case, we know that

$$\begin{aligned} E &= \frac{1}{2}mv^2 \\ E &\equiv \text{energy in joules} \\ m &\equiv \text{mass in kilograms} \\ v &\equiv \text{velocity in meters per second} \end{aligned}$$

3. **Use Algebra to Isolate the Unknown Quantity**

   We want to know the mass m, so we need to manipulate the equation we found in part 2 to get m by itself.

$$E = \frac{1}{2}mv^2$$

   Multiply on both sides by 2

$$2E = 2\frac{1}{2}mv^2$$

   The two in the numerator cancels with the two in the denominator

$$2E = mv^2$$

   Next divide both sides by v^2

$$\begin{aligned} \frac{1}{v^2}2E &= \frac{1}{v^2}mv^2 \\ \frac{2E}{v^2} &= m\frac{v^2}{v^2} \\ \frac{2E}{v^2} &= m(1) \\ \frac{2E}{v^2} &= m \end{aligned}$$

The last line is the relation we want

$$m = \frac{2E}{v^2}$$

4. **Use a Consistent System of Units**

In the statement of the problem, we were given a mixture of units: megatons, kilometers per second, etc. and we we need to make sense out of it all. In almost all cases, it's a good idea to turn everything into meters, kilograms, and seconds. So what do we need to do?

- Turn megatons into joules

$$50 \text{ megatons} \times \frac{1000 \text{ kilotons}}{1 \text{ megaton}} \times \frac{4.184 \times 10^{12} \text{ joules}}{1 \text{ kiloton}} = (50)(1000)(4.184 \times 10^{12}) \text{ joules}$$

So the final answer is that 50 megatons is equivalent to 2.0920×10^{17} joules. In terms of an equation

$$E = 2.0920 \times 10^{17} \text{ J}$$

- Turn kilometers per second into meters per second

$$11 \frac{\text{kilometers}}{\text{second}} \times \frac{1000 \text{ meters}}{1 \text{ kilometer}} = (11)(1000) \frac{\text{meters}}{\text{seconds}}$$

So the final answer is that 11 kilometers per second is equivalent to 11000 meters per second. In terms of an equation

$$v = 11000 \text{ m s}^{-1}$$

5. **Plug in the Numbers**

We now know that

$$m = \frac{2E}{v^2}$$

and that

$$\begin{aligned} E &= 2.0920 \times 10^{17} \text{ J} \\ v &= 11000 \text{ m s}^{-1} \end{aligned}$$

Plugging in the numbers will reveal that

$$m = \frac{2(2.0920 \times 10^{17})}{(11000)^2} = \frac{4.1840 \times 10^{17}}{121000000} = 3.4579 \times 10^9 \text{ kg}$$

This is the size of asteroid about 50 meters across.

6. **State the answer in a sentence**

An object that has a mass of 3.5×10^9 kg and is moving at 11 km s^{-1} will release 50 megatons of energy, the equal of the largest nuclear weapon ever detonated.